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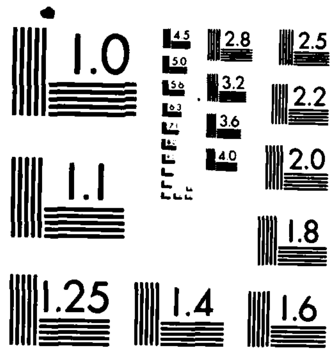

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MULTIPLE PROBABILITIES

by

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READINESS RESEARCH
GWU/IMSE Serial T-499/85
10 July 1985

THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Washington, DC 20052

Institute for Management Science and Engineering

Research Supported
by
Contract N00014-83-K-0216
Project NR 347-131
Office of Naval Research

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER GWU/IMSE/Serial T-499/85	2. GOVT ACCESSION NO. AD-A161 982	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RESULTS OF A SIMULATION STUDY TO MEASURE THE EFFECTIVENESS OF EMPIRICAL BAYES' ESTIMATES OF MULTIPLE PROBABILITIES		5. TYPE OF REPORT & PERIOD COVERED SCIENTIFIC 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) S. S. Brier, Z. Zacks, W. H. Marlow		8. CONTRACT OR GRANT NUMBER(s) N00014-83-K-0216
9. PERFORMING ORGANIZATION NAME AND ADDRESS GEORGE WASHINGTON UNIVERSITY INSTITUTE FOR MANAGEMENT SCIENCE AND ENGINEERING WASHINGTON, DC 20052		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS OFFICE OF NAVAL RESEARCH CODE 411 S&P ARLINGTON, VIRGINIA 22217		12. REPORT DATE 10 July 1985 13. NUMBER OF PAGES 15
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC SALE AND RELEASE; DISTRIBUTION UNLIMITED.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

BINARY DATA
CROSS-VALIDATION
EM ALGORITHM
EMPIRICAL BAYES ESTIMATES
READINESS EVALUATION

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Empirical Bayes' methods have been used by Brier, Zacks, and Marlow (1985) to estimate a set of parameters $\underset{\sim}{\theta}_1,\ldots,\underset{\sim}{\theta}_n$, where each $\underset{\sim}{\theta}_i$ is a k-dimensional vector consisting of the probabilities that a Marine combat unit will successfully complete requirements in each of k different categories. In this paper we assess the performance of the empirical Bayes' estimators by simulating $\underset{\sim}{\theta}_1,\ldots,\underset{\sim}{\theta}_n$ from a multivariate normal distribution and simulating the observed

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S/N 0102-014-6601

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sample counts from the binomial distribution. The empirical Bayes' estimators are compared to the sample proportion in terms of mean square error and absolute deviations. Using both criteria, the Bayes' estimators are shown to provide considerable improvement across a wide range of parameter values. The results suggest that empirical Bayes' estimates are useful in a variety of situations where a set of multivariate binomial probabilities are to be estimated.

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Abstract
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parameters

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Empirical Bayes' methods have been used by Brier, Zacks, and Marlow (1985) to estimate a set of parameters $\underset{\sim}{\theta}_1,\ldots,\underset{\sim}{\theta}_n$, where each $\underset{\sim}{\theta}_i$ is a k-dimensional vector consisting of the probabilities that a Marine combat unit will successfully complete requirements in each of k different categories. In this paper we assesses the performance of the empirical Bayes' estimators by simulating ~~$\underset{\sim}{\theta}_1,\ldots,\underset{\sim}{\theta}_n$~~ from a multivariate normal distribution and simulating the observed sample counts from the binomial distribution. The empirical Bayes' estimators are compared to the sample proportion in terms of mean square error and absolute deviations. Using both criteria, the Bayes' estimators are shown to provide considerable improvement across a wide range of parameter values. The results suggest that empirical Bayes' estimates are useful in a variety of situations where a set of multivariate binomial probabilities are to be estimated.

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1. Introduction

Brier, Zacks, and Marlow (1985) considered the problem of simultaneously estimating a set of vector parameters $\underset{\sim}{\theta}_1, \ldots \underset{\sim}{\theta}_n$, where $\underset{\sim}{\theta}_i$ is a vector of probabilities. The problem was motivated by the Marine Corps Combat Readiness Evaluation System (MCCRES), which is designed to measure the state of readiness of Marine combat units. In MCCRES units are tested to determine whether or not they can satisfactorily perform a number of requirements; the result of each test is classified as either satisfactory or unsatisfactory. The requirements, which typically number in the hundreds, fall into one of k categories and it is assumed that, for a given unit, the probability of satisfactorily completing a requirement is the same for all requirements in the same category. In MCCRES, the categorization of requirements is defined by the following six categories:

1. Advanced preparation and education

2. Combat service support and logistics

3. Mission planning and preparation

4. Command control and task organization

5. Execution

6. Information and communication

The available data consists of $\{N_{ij}\}$, $i=1,\ldots n$, $j=1,\ldots,k$, the number of tested requirements in the jth category applicable to the ith unit, and $\{X_{ij}\}$, the corresponding number of requirements that were satisfied. Brier, Zacks, and Marlow proposed using empirical Bayes' estimators of $\{\theta_{ij}\}$ as an improvement over the sample proportions $\{X_{ij}/N_{ij}\}$. The Bayes' estimators are based on the arcsin transformation of the proportions, $Y_{ij} = 2 \arcsin [(X_{ij} + 3/8)/(N_{ij} + 3/4)]^{1/2}$, and the corresponding transformation of the probabilities, $\eta_{ij} = 2 \arcsin \sqrt{\theta_{ij}}$. The assumptions of the Bayes model are that $\underset{\sim}{\eta}_i (\underset{\sim}{\eta}_i \equiv (\eta_{i1},\cdots\eta_{ik}))$, $i=1,\ldots n$ are independent $N_k\ (\mu,\Sigma)$ and that, conditional on η_{ij}, X_{ij} is binomially distributed and therefore Y_{ij} is distributed approximately as $N(\eta_{ij}, N_{ij}^{-1})$. These assumptions lead to Bayes' estimators of the form $\hat{\underset{\sim}{\eta}}_i = \underset{\sim}{Y}_i - A\ (\underset{\sim}{Y}_i - \underset{\sim}{\mu})$, where A is a function of Σ and $\{N_{ij}\}$.

Brier, Zacks, and Marlow (1985) discussed the appropriateness of the above assumptions. They obtained empirical Bayes estimators (EBE's) by replacing $\underset{\sim}{\mu}$ and Σ by their maximum likelihood estimates which were computed by applying the EM algorithm of Dempster, Laird and Rubin (1977). The model was applied to the data of Volume II of MCCRES and the EBE's were compared to the sample proportions as estimators of the true probabilities. Cross-validation techniques clearly demonstrated that the EBE's provided a substantial improvement over the sample proportions for the Volume II data.

The purpose of this study is to explore the performance of the EBE across a wider range of parameter values and sample sizes. To

accomplished this we used simulation techniques as described below. Although the degree of improvement obtained by the EBE's varies somewhat over the values of the parameters, the simulation study indicates that the EBE's are to be preferred, in general, to the sample proportions.

2. Results of the Simulation Study

In our simulation study we have fixed k to be six, corresponding to the MCCRES categorization of requirements. The number of units, n, was either 50 or 100. The number of requirements per category, N_{ij}, was constant over all (i,j), i.e., $N_{ij} = N$, and we considered three values of N in the study, N=5,25,50. The parameters of the distribution of $\underset{\sim}{\eta}$ are $\underset{\sim}{\mu}$ and $\underset{\sim}{\Sigma}$, the mean vector and covariance matrix, respectively. We set all six components of $\underset{\sim}{\mu}$ equal and considered $\mu = 1.57$, 2.09, and 2.50 in the study. Note that these means are in the arcsin scale and correspond to probabilities of .5, .75, and .9, respectively. With regard to Σ, we set all variance equal (σ^2) and all correlations equal (ρ). Our study fixed σ^2 at either .01 or .10 and ρ at either 0, .4, or .8.

Two criteria were used to compare the performance of the estimators. The First criteria is the total mean square error of estimation.

$\sum_{i=1}^{n} \sum_{j=1}^{6} (\hat{\eta}_{ij} - \eta_{ij})^2$. These measures are presented in Table 1.

Another measure of performance was obtained by examining the absolute error of estimation and tabulating, in each case, the proportion of errors that are greater than, .01, .05, .10, and .20, respectively. Table 2 presents the results of this comparison.

In addition to comparing the mean square errors of the ordinary estimates and the empirical Bayes' estimates, we also considered the mean square error of estimation of a third estimator.

$$\hat{\eta}_{ij}^{(s)} = \bar{Y}_i + \left(1 - \frac{4}{\sum_{j=1}^{6} N_{ij}\,(Y_{ij} - \bar{Y}_i)^2}\right)^+ (Y_{ij} - \bar{Y}_i) ,$$

where $\bar{Y}_i = (\sum_{j=1}^{6} N_{ij}\, Y_{ij})/(\sum_{j=1}^{6} N_{ij})$, and $a^+ \equiv \max\,(0,a)$. This estimator was suggested by James and Stein (1961) and was considered by Brier, Zacks, and Marlow. Table 1 demonstrates that $\hat{\eta}_{ij}^{(s)}$, the Stein estimator, is clearly inferior to the Bayes estimator and, in many instances, is inferior to the sample proportions. These findings are consistent with those of Brier, Zacks, and Marlow, and hence the Stein estimator will not be considered further.

An examination of Tables 1 and 2 indicates the clear superiority of the Bayes' estimates across the range of parameters. Even in the case where $\rho = 0$, in which the Bayes' estimates should provide the least amount of improvement, the Bayes estimates are almost always better than the ordinary proportions, especially in situations where the sample sizes are small.

A number of important aspects of the results should be noted. Although the performances of both estimators deteriorate as N becomes small, the relative advantage of the Bayes' estimators increases in small samples. This is to be expected since the Bayes' estimators "borrow strength" from other units and other category scores within the same unit. The relative performance of the Bayes' estimators also increases with the degree of correlation between category scores: when $\rho=.8$, even when N=50 (and the individual proportions contain an appreciable amount of information about the corresponding probabilities), the mean square error of the Bayes' estimates are, at worst, of the order of one-half that of the ordinary estimates. But even when $\rho = 0$, the Bayes' estimates are, at worst, comparable to the ordinary estimates. Finally, we note that 50 evaluations appears to provide a more than adequate size sample to estimate μ and Σ.

3. Conclusions

The results of this study strongly support the conclusion that empirical Bayes' estimators will provide considerable improvement over the sample proportions in situations where the category probabilities are positively correlated and the units are drawn from a moderately homogeneous population. Although we have only studied the behavior of the Bayes' estimators for k=6 categories, we would expect comparable results for larger values of k if the number of units, n, were increased to account for the extra parameters in $\underset{\sim}{\mu}$ and Σ that would require estimation.

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TABLE 1

Comparison of Mean Square Errors for Ordinary Estimates
Stein Estimates, and Bayes' Estimates

$\mu = 1.57$

Number of Evaluations	Number of Observations Per Category		$\sigma^2=.1$ $\rho=0$	$\sigma^2=.1$ $\rho=.4$	$\sigma^2=.1$ $\rho=.8$	$\sigma^2=.01$ $\rho=0$	$\sigma^2=.01$ $\rho=.4$	$\sigma^2=.01$ $\rho=.8$
50	5	Ordinary	13.46	12.62	14.36	15.48	14.89	14.20
		Stein	8.38	5.48	3.71	2.64	3.56	2.34
		Bayes	5.24	4.87	3.21	1.25	1.40	1.25
	25	Ordinary	2.71	2.34	2.53	2.74	2.70	3.11
		Stein	4.40	3.63	1.73	1.06	0.82	0.57
		Bayes	2.09	1.63	1.40	0.64	0.66	0.62
	50	Ordinary	1.27	1.30	1.27	1.57	1.53	1.61
		Stein	2.11	2.10	1.26	0.78	0.68	0.35
		Bayes	1.03	1.01	0.82	0.56	0.51	0.41
100	5	Ordinary	27.92	26.58	26.10	27.22	29.88	26.89
		Stein	15.89	11.19	7.03	5.36	6.86	3.93
		Bayes	9.71	8.46	6.46	1.85	2.44	2.04
	25	Ordinary	5.52	5.53	4.75	5.93	6.20	5.58
		Stein	8.13	5.95	3.08	1.99	2.04	1.18
		Bayes	3.82	3.50	2.33	1.36	1.33	1.06
	50	Ordinary	2.77	2.58	2.77	2.90	2.81	2.99
		Stein	4.60	4.63	2.54	1.81	1.15	0.79
		Bayes	2.23	2.07	1.73	1.16	0.77	0.85

TABLE 1 - (Cont'd)

$\mu = 2.09$

Number of Evaluations	Number of Observations Per Category		σ^2=.10 ρ=0	σ^2=.10 ρ=.4	σ^2=.10 ρ=.8	σ^2=.01 ρ=0	σ^2=.01 ρ=.4	σ^2=.01 ρ=.8
		Ordinary	11.46	12.29	10.12	10.43	11.72	11.46
	5	Stein	6.07	4.79	3.12	2.11	1.83	2.19
		Bayes	4.34	4.11	2.81	0.92	1.30	1.09
		Ordinary	1.91	2.09	2.05	2.04	2.10	2.20
50	25	Stein	4.00	2.73	1.02	0.83	0.68	0.45
		Bayes	1.60	1.48	0.86	0.48	0.49	0.40
		Ordinary	0.91	1.01	0.88	1.06	1.08	1.14
	50	Stein	2.19	2.29	1.03	0.60	0.41	0.29
		Bayes	0.79	0.91	0.55	0.43	0.38	0.29
		Ordinary	19.52	21.99	21.61	22.79	24.51	19.67
	5	Stein	11.40	9.27	5.43	3.98	4.24	3.19
		Bayes	7.23	6.43	4.90	1.54	1.90	1.49
		Ordinary	4.54	4.38	4.13	4.38	4.50	4.44
100	25	Stein	8.04	5.29	2.36	1.58	1.35	0.93
		Bayes	3.40	2.96	1.81	0.98	1.00	0.87
		Ordinary	2.20	1.95	2.19	2.19	2.17	2.06
	50	Stein	5.15	3.99	1.98	1.34	0.91	0.54
		Bayes	1.90	1.54	1.23	0.81	0.69	0.53

TABLE 1 - (Cont'd)

			$\mu = 2.50$					
			$\sigma^2=.1$			$\sigma^2=.01$		
Number of Evaluations	Number of Observations Per Category		$\rho=0$	$\rho=.4$	$\rho=.8$	$\rho=0$	$\rho=.4$	$\rho=.8$
50	5	Ordinary	5.00	4.63	5.48	5.63	6.05	5.09
		Stein	2.74	1.99	1.49	1.12	1.09	1.28
		Bayes	2.51	2.42	1.95	0.68	0.88	0.73
	25	Ordinary	1.04	1.23	0.97	1.27	1.08	0.87
		Stein	2.17	1.30	0.65	0.47	0.30	0.18
		Bayes	0.79	0.64	0.58	0.34	0.27	0.17
	50	Ordinary	0.49	0.58	0.57	0.52	0.58	0.64
		Stein	1.77	1.34	0.54	0.33	0.25	0.16
		Bayes	0.43	0.45	0.30	0.20	0.22	0.16
100	5	Ordinary	11.82	10.03	11.15	10.88	10.72	9.70
		Stein	5.87	4.51	2.88	2.09	2.14	1.53
		Bayes	5.10	5.06	3.51	1.33	1.29	1.11
	25	Ordinary	2.34	2.49	2.36	2.19	2.11	2.20
		Stein	1.69	3.35	1.46	0.70	0.62	0.49
		Bayes	4.04	1.68	1.21	0.41	0.41	0.39
	50	Ordinary	1.21	1.09	1.00	0.97	1.06	1.00
		Stein	3.65	2.61	1.09	0.55	0.46	0.29
		Bayes	1.03	0.82	0.61	0.37	0.33	0.26

TABLE 2

Distribution of Absolute Errors, for Ordinary Estimates and for Empirical Bayes' Estimates

(a) $\mu = 1.57$, $\sigma^2 = .10$

Number of Evaluations	Number of Observations Per Category	ρ	Proportions of Errors Greater Than: .01		.05		.10		.20	
	5	0	.98*	.96*	.84*	.74*	.68*	.46*	.38*	.11*
		.4	.95	.94	.77	.67	.59	.44	.33	.13
		.8	.98	.94	.86	.63	.68	.36	.40	.05
50	25	0	.89	.91	.58	.55	.31	.24	.03	.01
		.4	.92	.87	.56	.47	.26	.18	.03	.01
		.8	.89	.84	.55	.48	.27	.15	.04	.00
	50	0	.89	.85	.46	.39	.11	.08	.00	0
		.4	.87	.86	.46	.40	.13	.09	.00	0
		.8	.89	.86	.46	.33	.12	.05	0	0
	5	0	.95	.93	.81	.71	.64	.45	.36	.13
		.4	.97	.91	.83	.67	.65	.42	.36	.10
		.8	.96	.94	.82	.65	.63	.33	.35	.06
100	25	0	.91	.90	.59	.53	.30	.21	.04	.01
		.4	.92	.91	.62	.51	.28	.19	.04	.01
		.8	.90	.88	.57	.43	.26	.10	.02	.00
	50	0	.88	.85	.45	.42	.15	.10	.00	0
		.4	.88	.84	.44	.37	.13	.09	.00	.00
		.8	.88	.85	.48	.34	.44	.07	.00	0

*First entry in each column corresponds to the ordinary estimates, second entry corresponds to the Bayes' estimates.

TABLE 2 - (Cont'd)

(b) μ = 1.57, σ^2=.01

Number of Evaluations	Number of Observations Per Category	ρ	Proportion of Errors Greater Than: .01		.05		.10		.20	
		0	.99	.88	.89	.45	.68	.12	.41	.01
	5	.4	1.00	.85	.91	.41	.65	.14	.39	.00
		.8	.98	.88	.89	.43	.66	.11	.35	0
		0	.90	.85	.61	.28	.34	.03	.03	0
50	25	.4	.91	.79	.58	.29	.30	.03	.03	0
		.8	.92	.79	.64	.24	.36	.03	.04	0
		0	.90	.77	.52	.28	.18	.03	.00	0
	50	.4	.89	.81	.47	.23	.15	.01	.01	0
		.8	.87	.82	.52	.19	.17	.00	.00	0
		0	.98	.85	.88	.36	.66	.08	.35	0
	5	.4	.99	.88	.89	.42	.65	.12	.38	.00
		.8	.99	.85	.89	.36	.66	.08	.38	.00
		0	.92	.83	.62	.28	.29	.04	.05	0
100	25	.4	.92	.85	.63	.32	.34	.03	.05	0
		.8	.94	.83	.60	.25	.32	.00	.03	0
		0	.90	.81	.49	.26	.14	.02	.00	0
	50	.4	.89	.77	.43	.16	.15	.01	.00	0
		.8	.88	.77	.46	.19	.16	.01	.00	0

TABLE 2 - (Cont'd)

(c) $\mu = 2.09$, $\sigma^2 = .10$

Number of Evaluations	Number of Observations Per Category	ρ	Proportion of Errors Greater Than: .01		.05		.10		.20	
		0	.94	.95	.78	.72	.60	.42	.29	.08
	5	.4	.97	.93	.86	.74	.65	.44	.31	.07
		.8	.97	.90	.74	.64	.52	.38	.24	.03
		0	.89	.88	.50	.49	.23	.16	.01	.01
50	25	.4	.94	.88	.53	.46	.22	.15	.01	.01
		.8	.89	.84	.54	.33	.20	.06	.04	0
		0	.84	.86	.34	.29	.07	.05	0	0
	50	.4	.85	.87	.39	.36	.09	.08	0	0
		.8	.83	.82	.33	.23	.07	.01	0	0
		0	.95	.92	.78	.68	.55	.37	.27	.06
	5	.4	.96	.92	.80	.68	.60	.34	.27	.04
		.8	.97	.91	.81	.60	.59	.29	.30	.02
		0	.86	.88	.52	.47	.24	.19	.03	.01
100	25	.4	.90	.90	.56	.49	.23	.15	.02	.01
		.8	.90	.86	.53	.36	.23	.06	.02	0
		0	.83	.86	.38	.37	.10	.07	.00	0
	50	.4	.86	.80	.35	.30	.08	.06	.00	0
		.8	.85	.82	.40	.25	.10	.03	.00	0

TABLE 2 - (Cont'd)

(d) $\mu = 2.09$, $\sigma^2 = .01$

Number of Evaluations	Number of Observations Per Category	ρ	Proportion of Errors Greater Than: .01		.05		.10		.20	
		0	.96	.86	.78	.40	.57	.08	.31	0
	5	.4	.94	.89	.79	.46	.61	.11	.34	0
		.8	.96	.87	.84	.46	.63	.10	.31	0
		0	.93	.77	.54	.20	.22	.01	.01	0
50	25	.4	.94	.80	.60	.24	.21	.01	.01	0
		.8	.92	.83	.57	.17	.25	.00	.02	0
		0	.88	.76	.40	.20	.09	.00	0	0
	50	.4	.85	.84	.39	.16	.11	.00	0	0
		.8	.84	.77	.41	.11	.11	0	0	0
		0	.96	.84	.80	.32	.62	.04	.33	0
	5	.4	.98	.86	.81	.40	.63	.08	.36	0
		.8	.98	.85	.78	.31	.60	.04	.28	0
		0	.91	.81	.58	.23	.25	.01	.01	0
100	25	.4	.91	.81	.59	.21	.24	.02	.02	0
		.8	.93	.80	.57	.18	.25	.01	.01	0
		0	.86	.78	.41	.18	.08	.00	0	0
	50	.4	.85	.76	.41	.14	.10	.01	0	0
		.8	.85	.74	.41	.09	.08	0	0	0

TABLE 2 - (Cont'd)

(e) $\mu = 2.50, \ \sigma^2 = .10$

Number of Evaluations	Number of Observations Per Category	ρ	Proportion of Errors Greater Than: .01		.05		.10		.20	
		0	.91	.93	.62	.68	.38	.33	.10	.02
	5	.4	.88	.88	.60	.58	.33	.32	.08	.03
		.8	.91	.91	.65	.59	.40	.22	.14	.02
		0	.77	.86	.33	.28	.09	.05	.01	0
50	25	.4	.83	.84	.37	.26	.09	.03	.01	0
		.8	.79	.79	.32	.19	.09	.04	.01	.00
		0	.71	.76	.22	.17	.02	.02	0	0
	50	.4	.75	.75	.25	.17	.04	.02	0	0
		.8	.73	.74	.22	.11	.03	.00	0	0
		0	.92	.93	.69	.67	.42	.34	.14	.01
	5	.4	.93	.93	.67	.68	.38	.33	.11	.02
		.8	.90	.91	.64	.57	.40	.20	.11	.01
		0	.81	.85	.34	.30	.11	.06	.00	.00
100	25	.4	.83	.86	.37	.32	.12	.05	.00	.00
		.8	.82	.83	.36	.22	.13	.04	.00	0
		0	.72	.78	.25	.20	.04	.03	0	0
	50	.4	.73	.77	.22	.16	.03	.02	0	0
		.8	.75	.72	.20	.12	.02	.00	0	0

TABLE 2 - (Cont'd)

(f) $\mu = 2.50$, $\sigma^2 = .01$

Number of Evaluations	Number of Observations Per Category	ρ	Proportion of Errors Greater Than: .01		.05		.10		.20	
		0	1.00	.83	.94	.32	.49	.02	.09	0
	5	.4	1.00	.89	.95	.41	.51	.03	.11	0
		.8	.99	.90	.93	.37	.49	.01	.08	0
		0	.90	.76	.41	.16	.12	.00	.01	0
50	25	.4	.88	.72	.38	.11	.10	.00	.00	0
		.8	.85	.65	.38	.04	.06	0	0	0
		0	.80	.70	.25	.06	.01	0	0	0
	50	.4	.82	.72	.26	.06	.02	0	0	0
		.8	.82	.68	.26	.03	.03	0	0	0
		0	1.00	.88	.96	.34	.50	.01	.10	0
	5	.4	1.00	.86	.96	.33	.45	.00	.10	0
		.8	1.00	.84	.94	.25	.49	.01	.07	0
		0	.87	.72	.43	.05	.08	0	.00	0
100	25	.4	.87	.72	.40	.05	.08	.00	.00	0
		.8	.88	.71	.40	.05	.09	0	.00	0
		0	.80	.72	.20	.05	.02	0	0	0
	50	.4	.82	.70	.21	.02	.01	0	0	0
		.8	.79	.65	.21	.02	.01	0	0	0

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